澳門大學
UNIVERSIDADE DE MACAU
UNIVERSITY OF MACAU

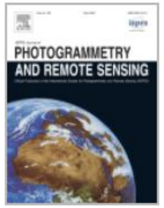
Fast and deterministic (3+1)DOF point set registration with gravity prior

翁霄羽
wengxiaoyu2009@163.com

ELSEVIER

ISPRS Journal of Photogrammetry and Remote Sensing

Volume 199, May 2023, Pages 118-132



Fast and deterministic (3+1)DOF point set registration with gravity prior

Xinyi Li [a], Yinlong Liu [b], Yan Xia [c], Venkatnarayanan Lakshminarasimhan [a], Hu Cao [a], Feihu Zhang [d], Uwe Stilla [c], Alois Knoll [a]

[a] Chair of Robotics, Artificial Intelligence and Real-time Systems, TUM School of Computation, Information and Technology, Technical University of Munich, 85748, Munich, Germany

[b] State Key Laboratory of Internet of Things for Smart City (SKL-IOTSC), University of Macau, 999078, Macao Special Administrative Region of China

[c] Photogrammetry and Remote Sensing, TUM School of Engineering and Design, Technical University of Munich, 80333, Munich, Germany

[d] School of Marine Science and Technology, Northwestern Polytechnical University, 710072, Xian, China

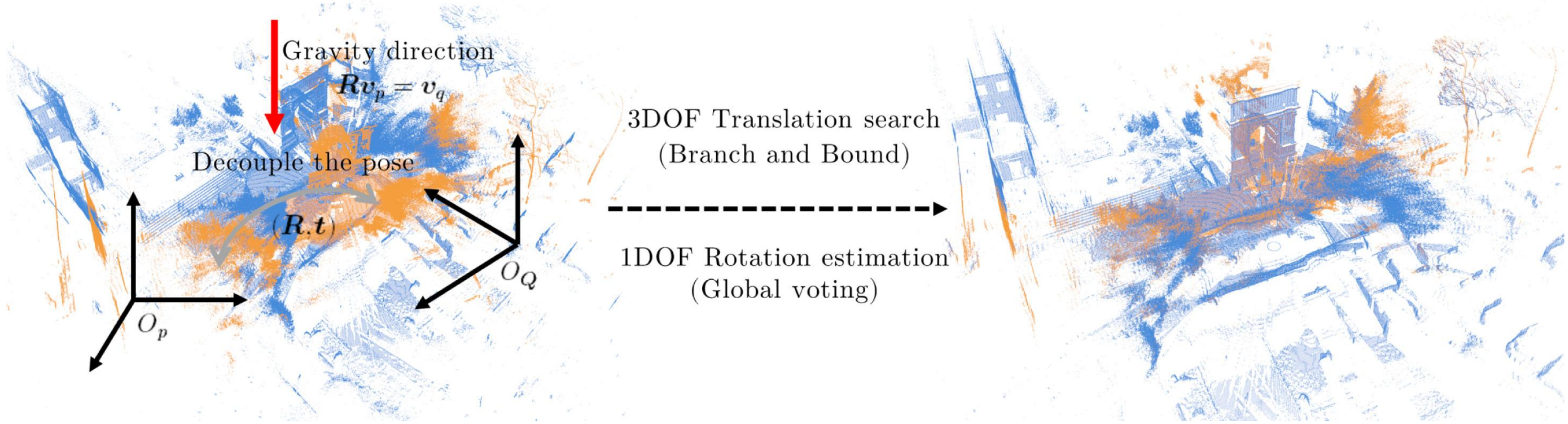
澳大

Introduction

1.The article decouple the 4DOF problem into two subproblems with gravity prior.

2.Propose a BnB-based optimization algorithm for the 3DOF translation search sub-problem.

3.Propose an efficient global voting algorithm for the 1DOF rotation estimation sub-problem.



Problem formulation—Inlier set maximization

The common 6DOF registration problem is intended to estimate the rigid transformation $\boldsymbol{T} \in SE(3)$, including rotation $\boldsymbol{R} \in SO(3)$ and translation $\boldsymbol{t} \in \mathbb{R}^3$

Inlier set maximization is not only robust but also easy to compute. The formalized optimization problem to be solved is:

$$\boldsymbol{T}^* = \underset{T \in SE(3)}{\arg\max}\, \mathcal{O}\big(\boldsymbol{T}(\mathcal{P}), \mathcal{Q}\big),$$

we adopt the angle-based criterion to measure the alignment of two point sets.

$$\boldsymbol{R}(\boldsymbol{p}_i + \boldsymbol{t}) = \boldsymbol{q}_i.$$

Then, given an arbitrary unit-norm vector $\boldsymbol{v} \in \mathbb{R}^3$, the angle between the direction vector represented by each of the two points and this unit-norm vector should be equal

$$\angle\big(\boldsymbol{R}(\boldsymbol{p}_i + \boldsymbol{t}), \boldsymbol{v}\big) = \angle(\boldsymbol{q}_i, \boldsymbol{v}).$$

Points $\boldsymbol{p}_i$ and $\boldsymbol{q}_i$ are considered as aligned (an inlier) only if $\left|\angle\big(\boldsymbol{R}(\boldsymbol{p}_i + \boldsymbol{t}), \boldsymbol{v}\big) - \angle(\boldsymbol{q}_i, \boldsymbol{v})\right| \leq \zeta.$ ζ is the angle-based inlier threshold.

Problem formulation—4DOF registration

With the aid of known gravity directions provided by IMUs, the 6DOF registration problem is reduced to 4DOF.

The constraint of the gravity direction is given by:

$$\boldsymbol{R}\boldsymbol{v}_p = \boldsymbol{v}_q,$$

The solution to this equation is:

$$\boldsymbol{R} = \boldsymbol{R}(\theta, \boldsymbol{v}_q) \cdot \boldsymbol{R}_{\boldsymbol{v}_p}^{\boldsymbol{v}_q},$$

The 4DOF registration problem is given by:

$$\theta^*, \boldsymbol{t}^* = \underset{\theta \in [-\pi,\pi], \boldsymbol{t} \in \mathbb{R}^3}{\arg\max} \sum_{i=1}^{N} \mathbb{I}\Big(\Big|\angle\big(\boldsymbol{R}(\boldsymbol{p}_i + \boldsymbol{t}), \boldsymbol{v}\big) - \angle(\boldsymbol{q}_i, \boldsymbol{v})\Big| \le \zeta\Big),$$

Where I is an indicator function that returns 1 if the input condition is true and 0 otherwise

Problem formulation—4DOF transformation decoupling by gravity direction

If we consider the known gravity direction $\boldsymbol{v}_q$ to be the arbitrary unit-norm vector $\boldsymbol{v}$, the objective function of our problem is:

$$E(\boldsymbol{R}, \boldsymbol{t}|\mathcal{K}, \zeta) = \sum_{i=1}^{N} \mathbb{I}\left(\left|\angle\left(\boldsymbol{R}(\boldsymbol{p}_i + \boldsymbol{t}), \boldsymbol{v}_q\right) - \angle(\boldsymbol{q}_i, \boldsymbol{v}_q)\right| \leq \zeta\right),$$

where $\boldsymbol{R}$ represents the 1DOF rotation, and K represents the set of candidate correspondences.

According to $\boldsymbol{R}\boldsymbol{v}_p = \boldsymbol{v}_q$, we have:

$$\begin{aligned}\angle\left(\boldsymbol{R}(\boldsymbol{p}_i + \boldsymbol{t}), \boldsymbol{v}_q\right) &= \angle\left(\boldsymbol{R}(\boldsymbol{p}_i + \boldsymbol{t}), \boldsymbol{R}\boldsymbol{v}_p\right) \\ &= \angle(\boldsymbol{p}_i + \boldsymbol{t}, \boldsymbol{v}_p).\end{aligned}$$

we can then define $\gamma_i = \angle(\boldsymbol{q}_i, \boldsymbol{v}_q)$. Thus the objective function can be rewritten as

$$E(\boldsymbol{t}|\mathcal{K}, \zeta) = \sum_{i=1}^{N} \mathbb{I}\left(\left|\angle(\boldsymbol{p}_i + \boldsymbol{t}, \boldsymbol{v}_p) - \gamma_i\right| \leq \zeta\right).$$

To sum up, we utilize the constraint of known gravity direction to decouple the 4DOF consensus maximization problem into 3DOF translation and 1DOF rotation sub-problems.

Decoupled 4DOF point set registration—Global translation search

In order to bound the objective function on an arbitrary branch, the bound for angle $\angle(\dot{\boldsymbol{p}} + \boldsymbol{t}, \boldsymbol{v}_p)$ needs to be found based on the uncertainty cube $C^{\dot{\boldsymbol{p}}}_{\boldsymbol{t}}$.

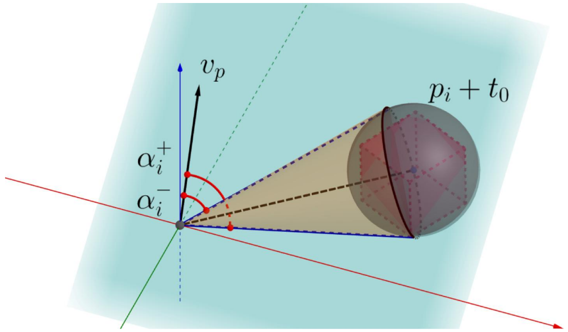
We then define α_i^0 as the middle point of the interval $[\alpha_i^-, \alpha_i^+]$ and can obtain:

$$\alpha_i^0 = (\alpha_i^- + \alpha_i^+)/2$$
$$= \angle(\boldsymbol{p}_i + \boldsymbol{t}_0, \boldsymbol{v}_p).$$

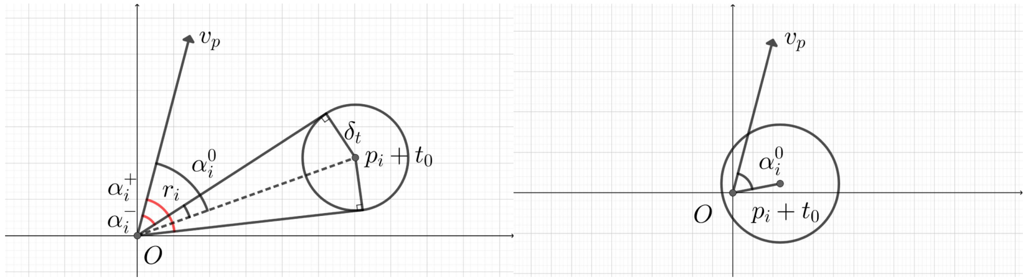
Moreover, we define r_i as the radius of the interval $[\alpha_i^-, \alpha_i^+]$,

there is another case in which the relaxed circumsphere intersects with the origin, such that ri is π

$$r_i = \begin{cases} \arcsin(\delta_t / \|\boldsymbol{p}_i + \boldsymbol{t}_0\|), & \text{if } \delta_t \le \|\boldsymbol{p}_i + \boldsymbol{t}_0\| \\ \pi, & \text{otherwise.} \end{cases}$$



Geometric principle of the proposed upper and lower bounds for BnB. The uncertainty cube $C_t^{p_i}$ is the red cube. The center of the cube is $\dot{\boldsymbol{p}} + \boldsymbol{t}_0$. The light gray sphere is the relaxed circumsphere of the cube. The bound for the uncertain angle $\dot{\alpha}$ is $[\alpha_i^-, \alpha_i^+]$.



Decoupled 4DOF point set registration—Global translation search

Finally, we derive the upper bound function on the sub-branch B ⊂ R^3 as follows:

$$\overline{E}(\mathbb{B}) = \sum_{i=1}^{N} \mathbb{I}\left(|\alpha_i^0 - \gamma_i| - r_i \leq \zeta\right).$$

In addition, the lower bound function can be:

$$\underline{E}(\mathbb{B}) = \sum_{i=1}^{N} \mathbb{I}\left(|\alpha_i^0 - \gamma_i| \leq \zeta\right).$$

Lemma 1. For any translation sub-branch B ⊂ R3 with center $\boldsymbol{t}_0$ and radius ρ_t , the upper bound and lower bound of the objective function (9) can be chosen as $\overline{E}$(B) and $\underline{E}$(B) from Eqs respectively.

The BnB-based 3DOF translation search algorithm is outlined in Algorithm 1 according to the proposed lower and upper bound in Lemma 1.

Decoupled 4DOF point set registration—Global translation search

Algorithm 1 BnB for 3DOF translation search (solution to problem (9))

Require: Solution domain $\mathbb{B}$, Inlier threshold ζ, Set of candidate correspondences $\mathcal{K} = \left\{(\boldsymbol{p}_i, \boldsymbol{q}_i)\right\}_{i=1}^{N}$, and Gravity directions $\boldsymbol{v}_p, \boldsymbol{v}_q$.

Ensure: Globally optimal solution $\boldsymbol{t}^*$.

1: Let ξ be the list of sub-branches, initialize $B_0 = \mathbb{B}$, $\xi = \{B_0\}$.
2: Define function $\delta(B)$ returns the center point of sub-branch B.
3: Initialize $LB = 0$, and $UB = N$.
4: **while** $UB - LB > 0$ **do**
5: Select a sub-branch B with the maximum upper bound from ξ, i.e., $B = \arg\max \overline{E}(B_k)$, $B_k \in \xi$. Then split B into eight sub-branches $S(B) = \{B_1, \dots, B_8\}$.
6: Delete B from ξ, and add $\{B_1, \dots, B_8\}$ to ξ.
7: Update $UB = \max \overline{E}(B_k)$, $B_k \in \xi$.
8: Update $LB = \max\left\{LB, \underline{E}(B_k)\right\}$ with $B_k \in \xi$. Meantime, if $\underline{E}(B_k) > LB$, set $\boldsymbol{t}^* = \delta(B_k)$.
9: Delete B_k from ξ with $\overline{E}(B_k) < LB$, $B_k \in \xi$.
10: **end while**
11: **return** $\boldsymbol{t}^*$

Decoupled 4DOF point set registration—Global rotation estimation

The geometric intuition is shown in Fig. 5. Moreover, the rotation $\boldsymbol{R}_{\boldsymbol{v}_p}^{\boldsymbol{v}_q}$ that aligns $\boldsymbol{v}_p$ to $\boldsymbol{v}_q$ with the minimum geodesic motion is:

$$\boldsymbol{R}_{\boldsymbol{v}_p}^{\boldsymbol{v}_q} = \exp\big(\rho[\boldsymbol{v}_m]_\times\big),$$

where ρ= arccos($\boldsymbol{v}_p \cdot \boldsymbol{v}_q$), and $\boldsymbol{v}_m = \frac{\boldsymbol{v}_p \times \boldsymbol{v}_q}{\|\boldsymbol{v}_p \times \boldsymbol{v}_q\|}$.

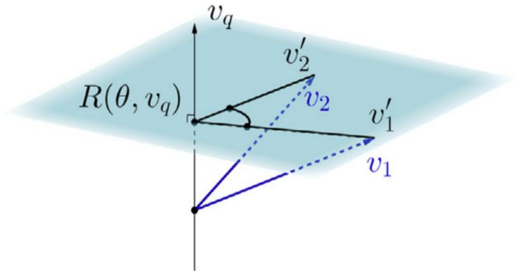


Fig. 5. Geometric intuition for angle θ, which is the angle between the projections of two vectors $\boldsymbol{v}_1 = \boldsymbol{R}_{\boldsymbol{v}_p}^{\boldsymbol{v}_q} \cdot (\boldsymbol{p}_i + \boldsymbol{t})$ and $\boldsymbol{v}_2 = \boldsymbol{q}_i$. The projections $\boldsymbol{v}_1'$ and $\boldsymbol{v}_2'$ are on the plane with $\boldsymbol{v}_q$ as the normal.

Overall, the 4DOF correspondence-based point set registration problem can be easily solved by running the 3DOF translation search in Algorithm 1 and the 1DOF rotation estimation in Algorithm 2.

Algorithm 2 Global voting for 1DOF rotation estimation (solution of problem (28))

Require: Globally optimal solution $\boldsymbol{t}^*$, Set of candidate correspondences $\mathcal{K} = \{(\boldsymbol{p}_i, \boldsymbol{q}_i)\}_{i=1}^N$, and Gravity directions $\boldsymbol{v}_p, \boldsymbol{v}_q$.

Ensure: Global solution θ^*.

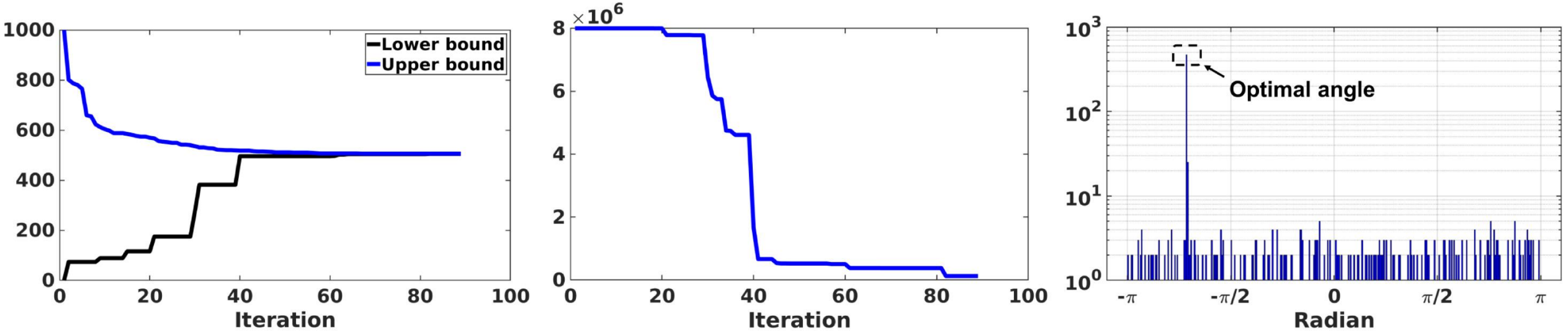
1: Calculate $\boldsymbol{R}_{\boldsymbol{v}_p}^{\boldsymbol{v}_q}$ according to Equation (31).

2: Calculate angle θ_i, $i = 1, \ldots, N$ for each candidate correspondence in $\mathcal{K}$.

3: Operate histogram voting in the interval $[-\pi, \pi]$ for angle $\theta_i, i = 1, \ldots, N$.

4: **return** θ^*

Experiments



(a) Convergence curve of proposed BnB (b) Remaining volume of translation domain (c) Global voting for rotation estimation

Efficient translation search by the proposed BnB and global voting for rotation estimation.

It is evident that the gap between the lower and upper bounds is converging to zero, and the proposed method converges to the optimal solution after dozens of iterations. The remaining volume of the translation domain decreases rapidly until convergence

Experiments

Controlled experiments with different outlier rates and noise levels

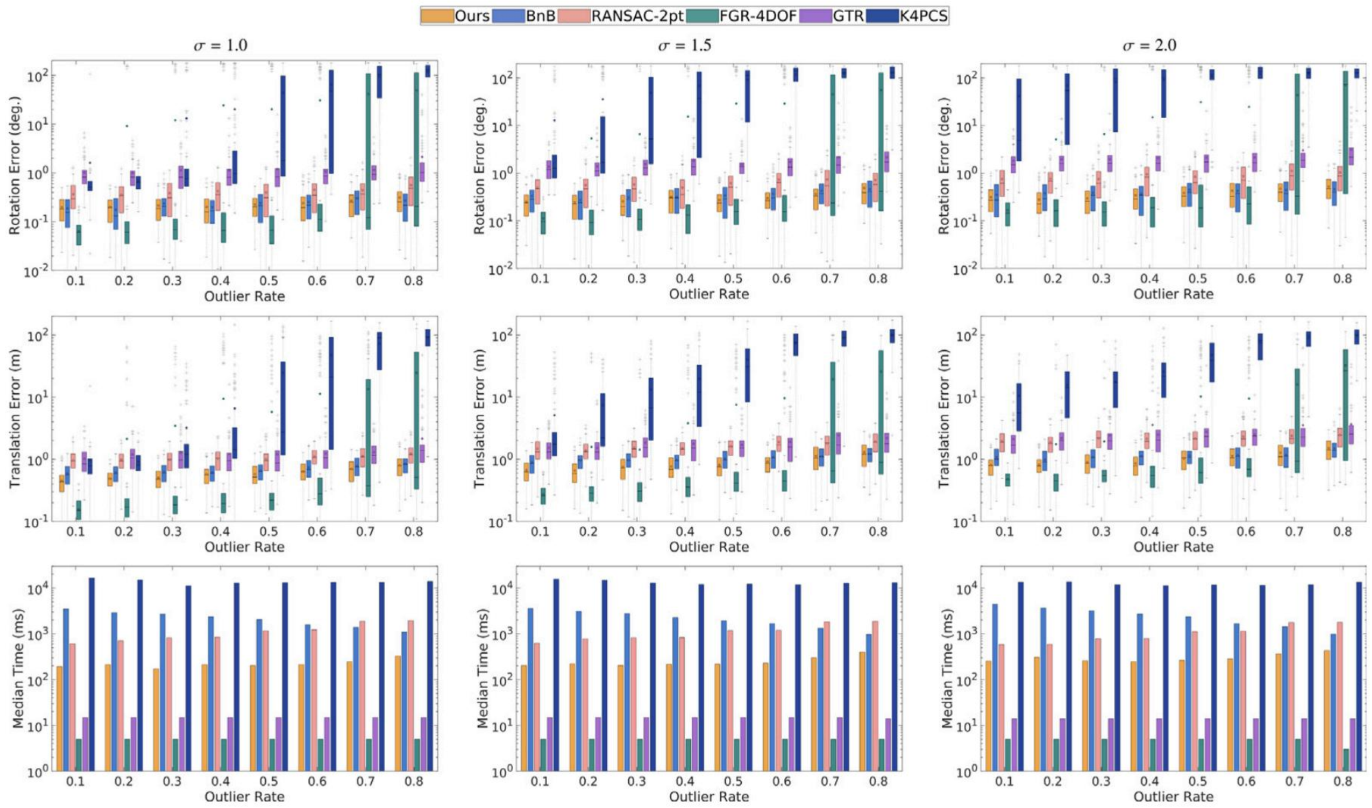


Fig. 8. Controlled experiments with normal ($\sigma = 1.0$) and high ($\sigma = 1.5, 2.0$) noise levels; in each group, the outlier rate is $\eta = \{0.1, 0.2, \dots, 0.8\}$. The results include rotation error, translation error, and median time.

Comparison of the rotation error and translation error shows that Ours and BnB perform well under all outlier rates. Moreover, in most cases, the average errors of Ours are smaller than that of all methods except FGR-4DOF.

澳大

Experiments

Controlled experiments with different numbers of correspondence

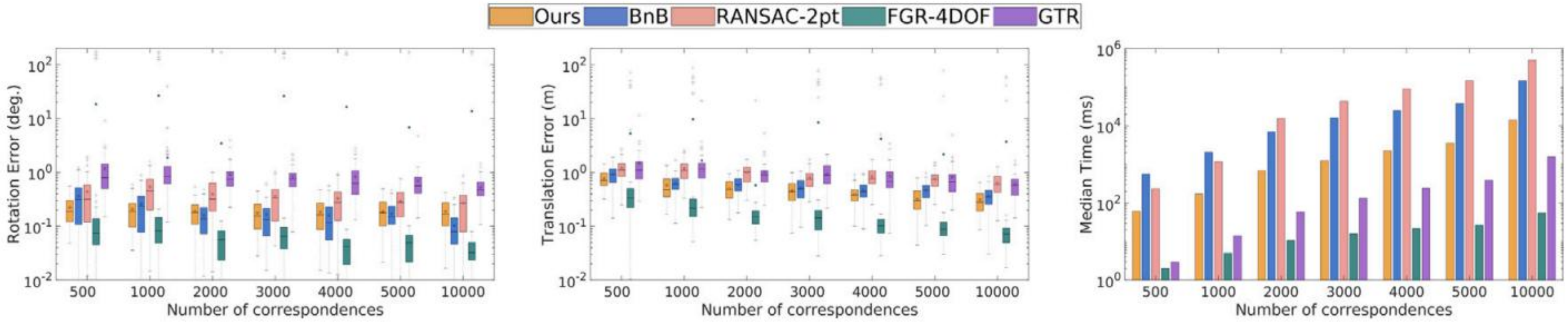


Fig. 9. Controlled experiments with different numbers of correspondences ($N = \{500, 1000, \ldots, 10000\}$). The outlier rate is $\eta = 0.5$ and the noise level is $\sigma = 1.0$. The results include rotation error, translation error, and median time.

For the deterministic methods, it can be observed that the accuracy of Ours is higher than BnB and lower than FGR-4DOF. However, the average rotation and translation errors of FGR-4DOF are the highest. These experiment results illustrate the superiority of our proposed method in terms of accuracy and efficiency.

Experiments

Robustness to gravity direction biases

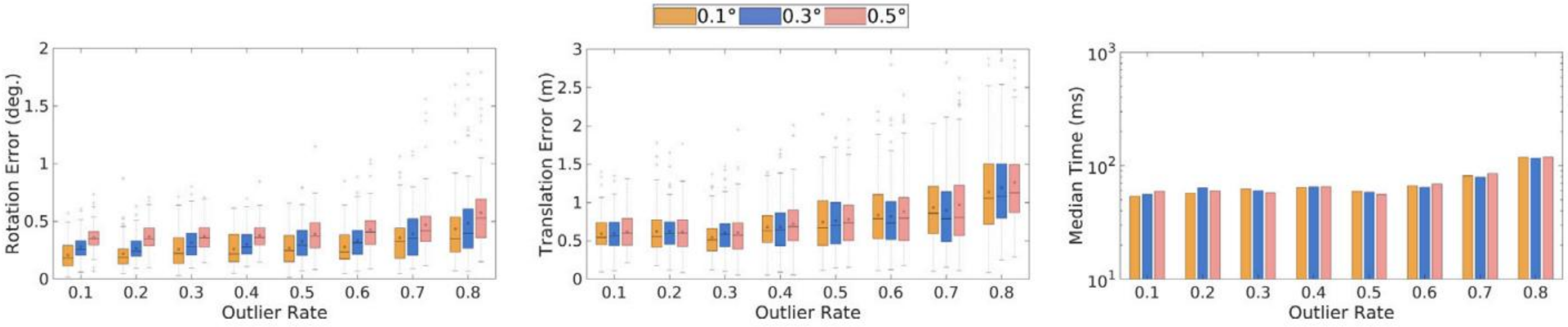


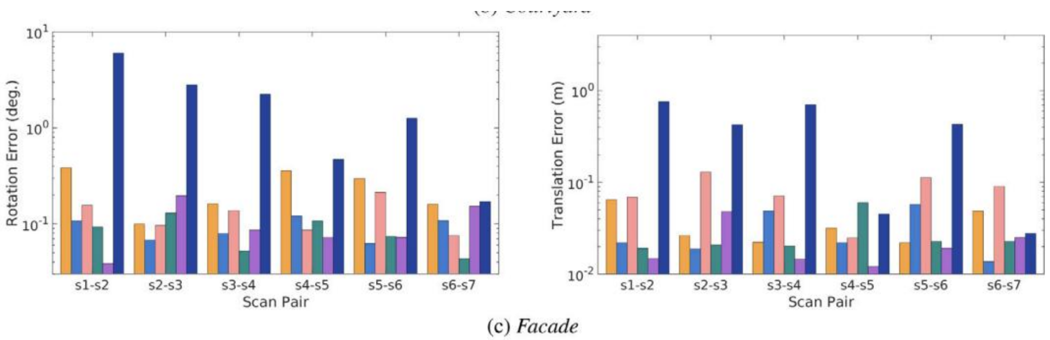
Fig. 10. Controlled experiments with different biased angles $(0.1°, 0.3°, 0.5°)$ in gravity directions and different outlier rates $\eta = \{0.1, 0.2, \ldots, 0.8\}$. The number of correspondences is $N = 500$, and the noise level is $\sigma = 1.0$. The results include rotation error, translation error, and median time.

As can be seen, different biased angles have a certain effect on the rotation error, but have little effect on the translation error. Overall, these experiments prove that our proposed method is robust to the biased angle in gravity directions and remains robust to outliers when gravity directions are biased

Experiments

Real-world data experiments

ETH dataset experiments



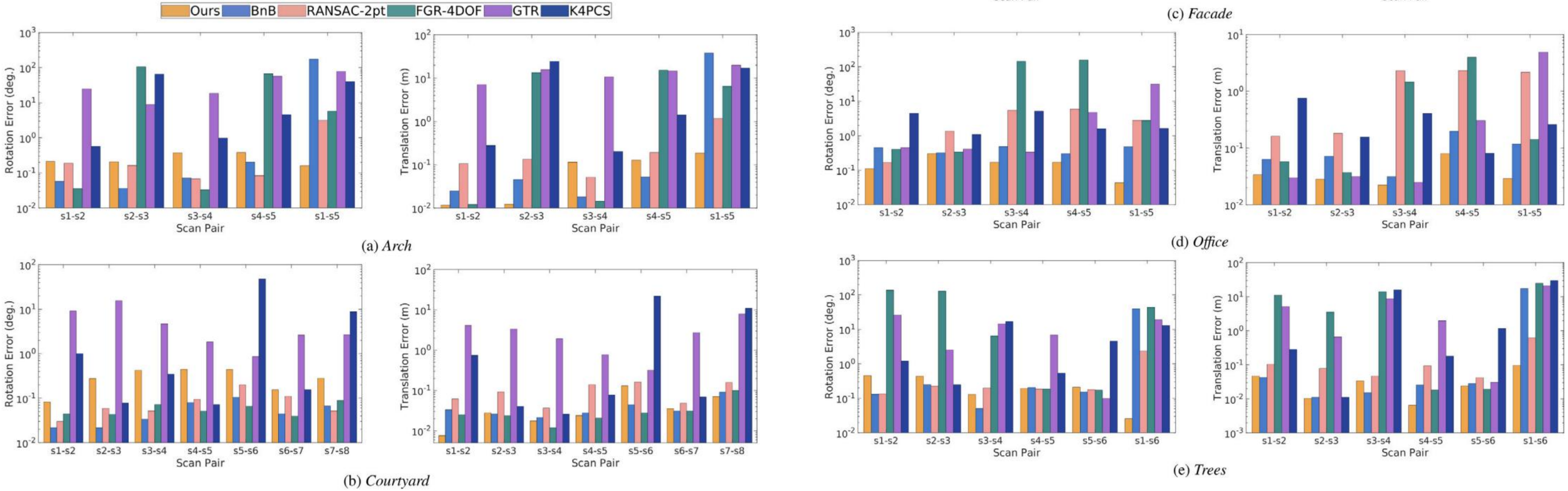


Fig. 12. Rotation error and translation error of all registration methods on the ETH dataset. Subfigures (a)–(e) show the registration results of each scan pair for Arch, Courtyard, Facade, Office, and Trees.

In most cases, the errors of Ours are not the lowest, since only rough gravity directions were employed in Ours. However, viewed overall, the errors of Ours are acceptable in practice in all cases.

Experiments

A9 dataset experiments

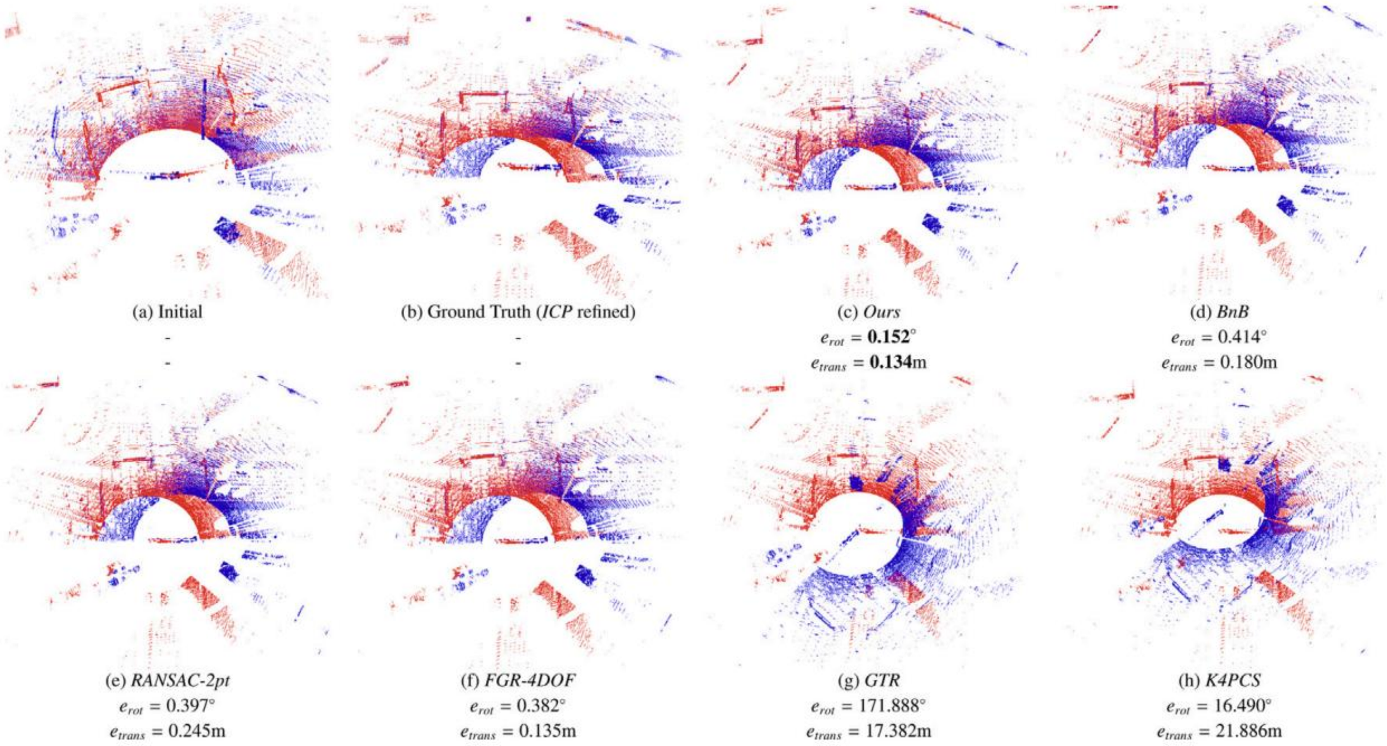


Fig. 14. Qualitative and quantitative results for one selected pair of scans (p1) on the A9 dataset. Each subplot contains the source point set (blue), target point set (red), and registration errors for (a) Initial, (b) Ground Truth, (c) *Ours*, (d) *BnB*, (e) *RANSAC-2pt*, (f) *FGR-4DOF*, (g) *GTR*, and (h) *K4PCS*. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

The qualitative and quantitative results for one selected pair of scans are shown in Fig. 14. The best experimental results were obtained by the proposed method.

澳大

Experiments

The proposed method achieved the best accuracy in the majority of cases. BnB achieved the second-best accuracy.

Table 3
Rotation error (°) for each pair of scans on the A9 dataset.

Dataset	*Ours*	*BnB*	*RANSAC-2pt*	*FGR-4DOF*	*GTR*	*K4PCS*
p1	**0.152**	0.414	0.397	0.382	171.888	16.490
p2	**0.144**	1.602	0.625	0.534	6.227	6.500
p3	**0.033**	0.641	0.401	0.510	1.452	123.891
p4	**0.159**	0.388	0.629	13.244	9.430	38.320
p5	0.438	**0.399**	0.404	3.075	131.941	175.362
p6	**0.138**	1.790	6.096	9.828	41.421	0.775

Table 4
Translation error (m) for each pair of scans on the A9 dataset.

Dataset	*Ours*	*BnB*	*RANSAC-2pt*	*FGR-4DOF*	*GTR*	*K4PCS*
p1	**0.134**	0.180	0.245	0.135	17.382	21.886
p2	**0.289**	0.590	4.076	0.337	2.144	0.588
p3	0.257	0.306	**0.215**	0.227	0.383	32.797
p4	**0.280**	0.342	0.453	11.699	3.004	24.091
p5	0.233	0.757	**0.207**	0.835	33.878	39.628
p6	**0.242**	1.019	33.186	10.170	9.477	0.433

The running times of each method are shown in Table 5.

Table 5
The left part is detailed information about the A9 dataset. The right part is the registration running time (ms) for each pair of scans on the A9 dataset.

Dataset	Number of points (10^4)	Number of key-points	Number of correspondences	*FMP*	*Ours*	*BnB*	*RANSAC-2pt*	*FGR-4DOF*	*GTR*	*K4PCS*
p1	11.35-10.88	318-304	1612	32	4.4	64	2496	5	46	690
p2	11.35-10.90	318-323	1643	34	6.7	78	4055	8	38	552
p3	11.36-10.86	301-334	1543	31	9.3	127	1262	9	35	544
p4	11.34-10.91	289-323	1607	27	8.4	85	1522	5	42	380
p5	11.37-10.89	319-288	1571	36	12.6	163	1516	7	41	514
p6	11.42-10.87	316-334	1654	34	27.6	550	3898	7	42	806

The proposed method achieved the best accuracy in the majority of cases. BnB achieved the second-best accuracy. Our proposed method is about 10 to 20 times faster than 4DOF BnB. This demonstrates the effectiveness of the decomposition mechanism in our proposed method.

Conclusion

1.In this paper, we present a fast and deterministic method for solving the 4DOF correspondence-based point set registration problem.

2.Leverage the known gravity directions to decouple the joint 4DOF pose into the sequential 3DOF translation and 1DOF rotation.

3.Proposed a BnB-based consensus maximization method for the 3DOF translation search sub-problem and derive the novel geometrical lower and upper bound functions for the global search.

4. Present an efficient global voting method based on geometrical principles for the 1DOF rotation estimation sub-problem.

5.Extensive experiments showed the superiority of our proposed method in terms of accuracy and efficiency compared to several existing methods.

Thank You!

Avenida da Universidade, Taipa, Macau, China
Tel : (853) 8822 8833 Fax : (853) 8822 8822
Email : info@um.edu.mo Website : www.um.edu.mo

um 澳大