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Accurate Rotation and Correspondence Search

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Calculation of rotation axis

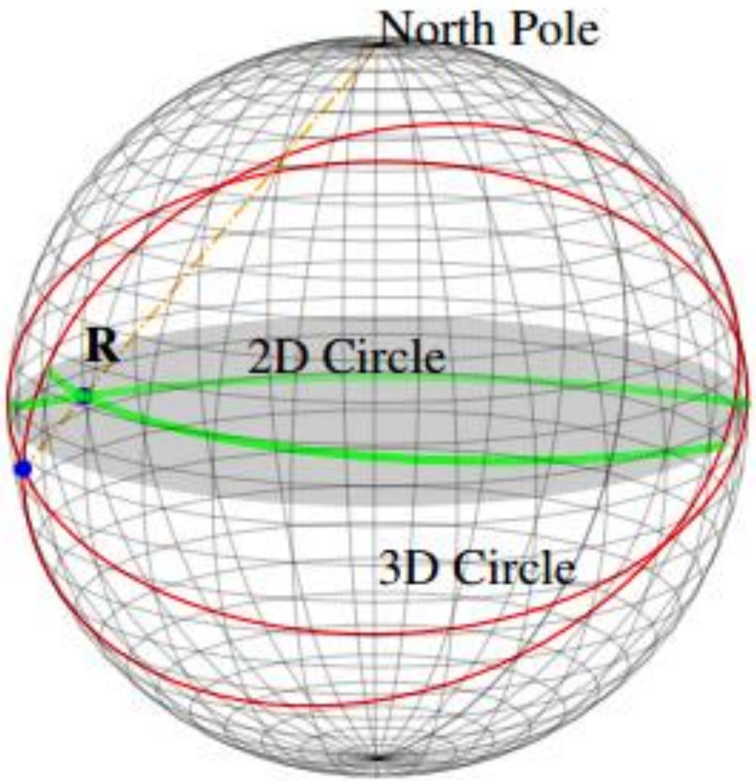
Known (x_i, y_i) is a pair of inner point, R is a rotation matrix, and b is an axis of rotation, so, $y_i = R^T x_i$ and R^Tb=b. Let $v_i = y_i - x_i$, then v_i^Tb = $(y_i - x_i)^T$b = $(y_i - Rx_i)^T$ b = 0.

Calculation of rotation axis

Let $v_i = (\alpha_i, \beta_i, \gamma_i)$, $\boldsymbol{b}$=(a^*, b^*, c^*)

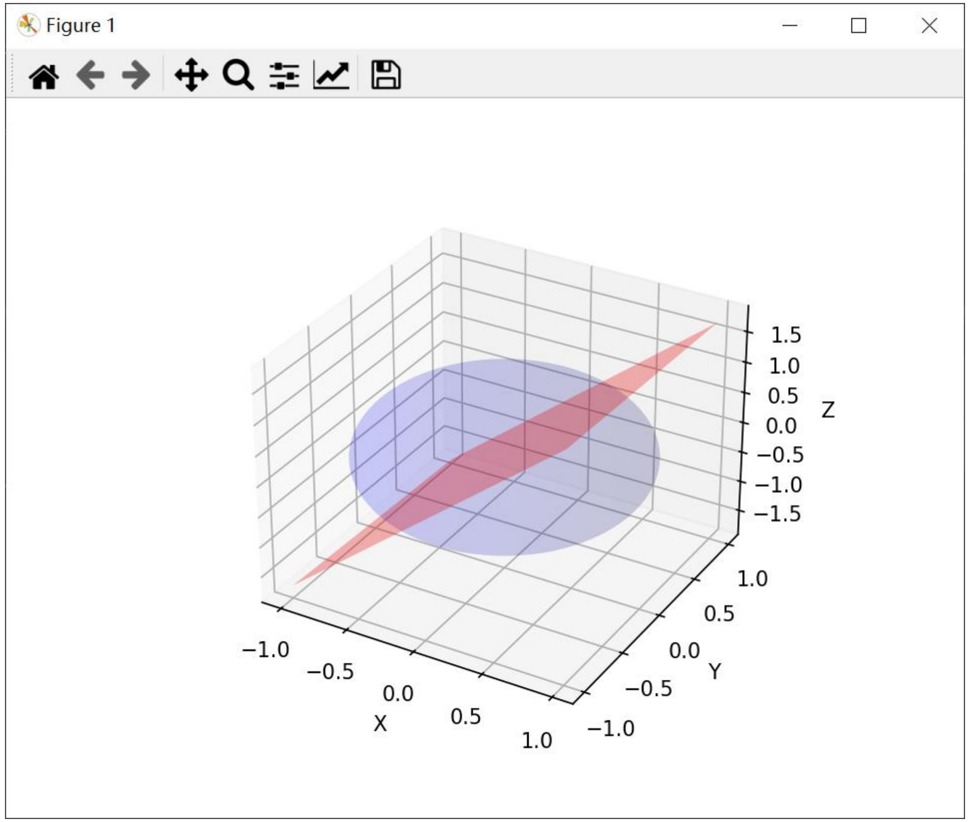
$$\alpha_i^2 + \beta_i^2 + \gamma_i^2 = 1$$

$$\begin{bmatrix} \alpha_1 & \beta_1 & \gamma_1 \\ \vdots & \vdots & \vdots \\ \alpha_n & \beta_n & \gamma_n \end{bmatrix} \begin{bmatrix} a^* \\ b^* \\ c^* \end{bmatrix} = 0$$

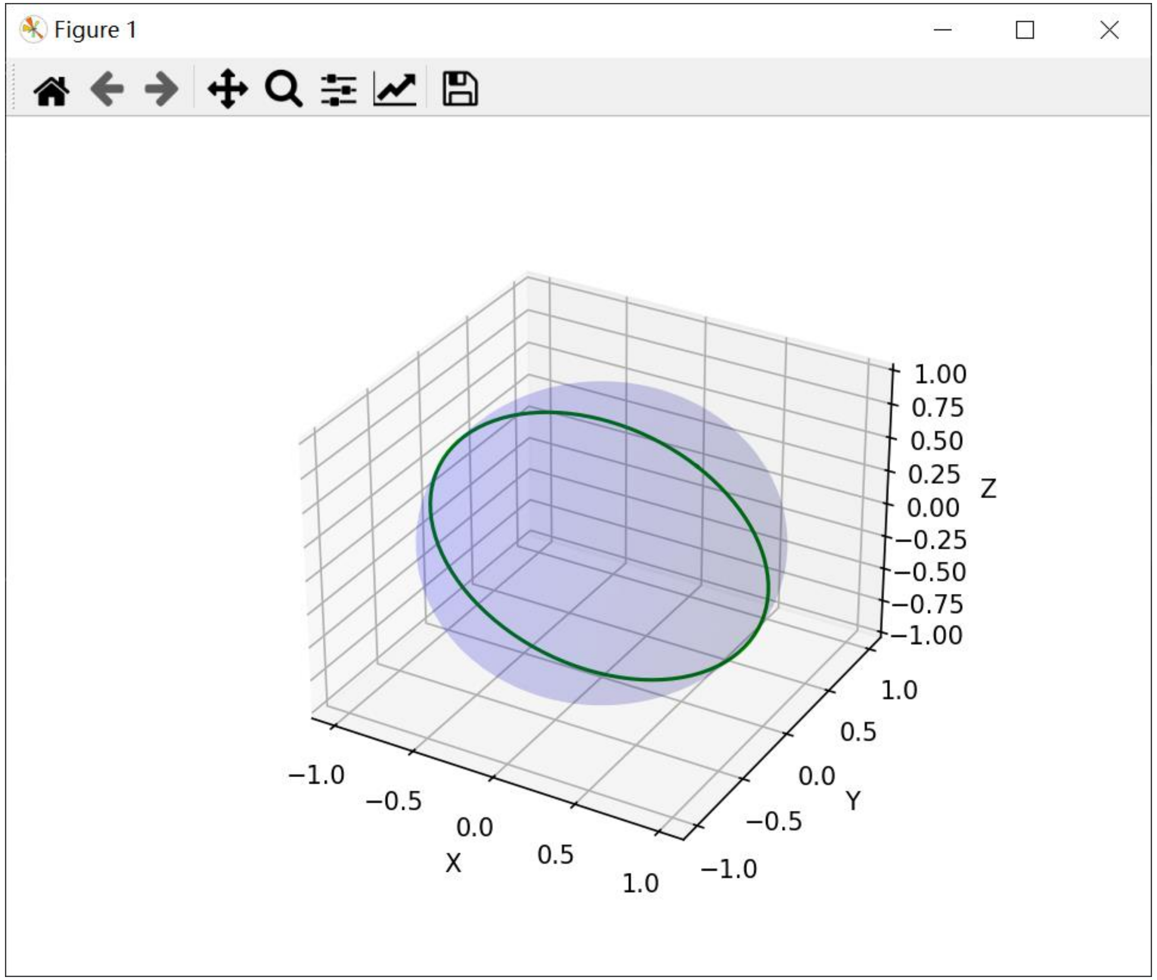


Calculation of rotation axis

$$\begin{cases} AX + BY + CZ + D = 0 \\ X^2 + Y^2 + Z^2 = 1 \end{cases}$$



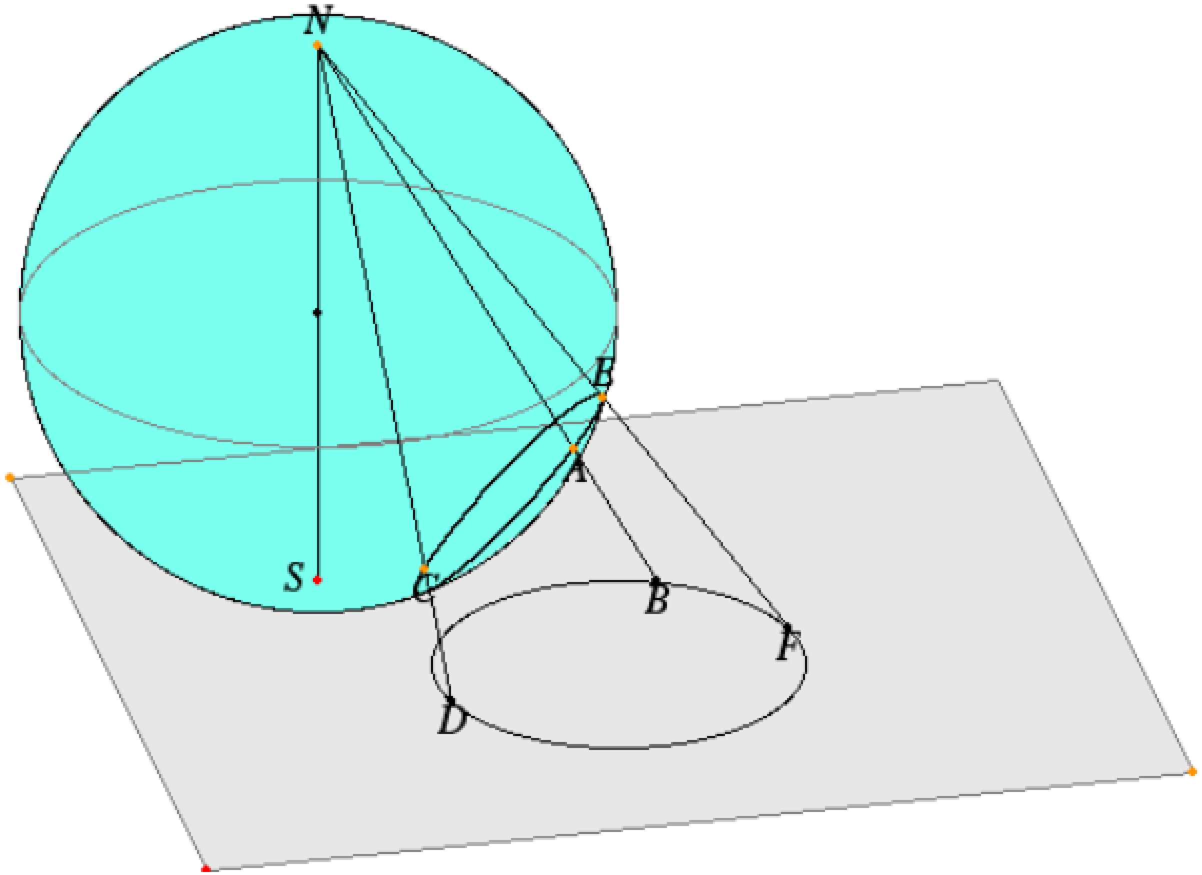
Calculation of rotation axis



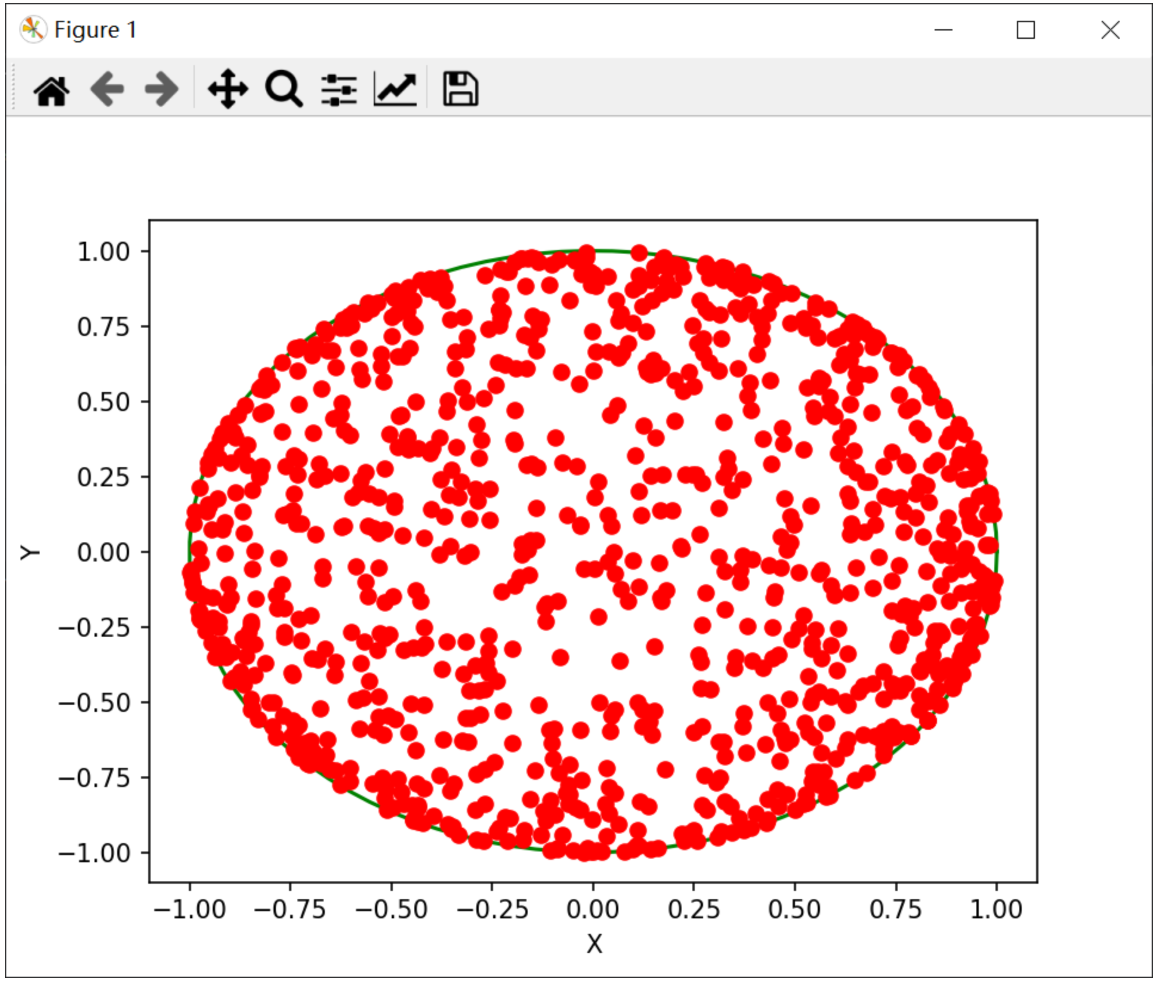
Calculation of rotation axis

$$(X,Y,Z) = \left(\frac{2x}{x^2+y^2+1}, \frac{2y}{x^2+y^2+1}, \frac{x^2+y^2-1}{x^2+y^2+1}\right).$$

$$\left(x + \frac{A}{C+D}\right)^2 + \left(y + \frac{B}{C+D}\right)^2 = \frac{A^2+B^2}{(C+D)^2} + \frac{C-D}{C+D}.$$



Calculation of rotation axis



Calculation of rotation angle

It is obtained by Rodrigues's Formula:
$R = \cos(\theta)I + (1 - \cos(\theta))nn^T + \sin(\theta)Skew(n)$

$$Skew(v) = \begin{bmatrix} 0 & -v_z & v_y \\ v_z & 0 & -v_x \\ -v_y & v_x & 0 \end{bmatrix}$$

$$Rx_i = y_i \quad [\cos(\theta)I + (1 - \cos(\theta))nn^T + \sin(\theta)Skew(n)]x_i = y_i$$

Estimated rotation matrix

$$R = \cos(\theta) I + (1 - \cos(\theta)) n n^T + \sin(\theta) Skew(n)$$

The judgment of the inliers

$$\left| \, \| Rm_k \| - \| n_k \| \, \right| = \left| \, \| m_k \| - \| n_k \| \, \right| \leq \mu_t$$

ground-truth

Firstly, we are given a set of observations x_i ,And then randomly generate a rotation matrix R, Finally, the target value is calculated from this rotation matrix y_i .The rotation matrix is the true rotation matrix R.

The error between the ground-truth and the estimated rotation matrix

$$e_R = \arccos(\frac{1}{2}(Tr(R_0^T R_t) - 1))$$

$$Tr(R_0^T R_t) = 1 + 2\cos(\Delta\theta)$$

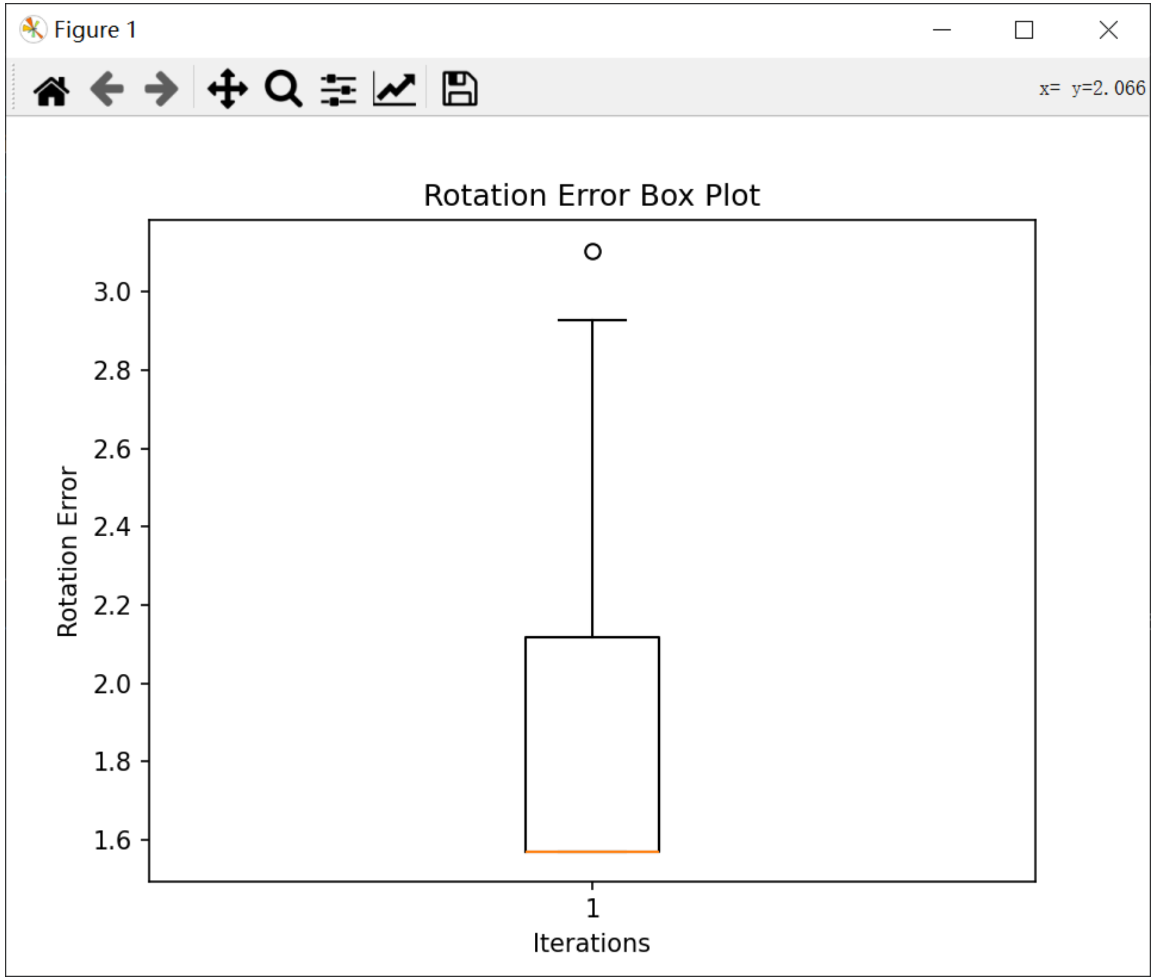
$$\Delta\theta = \arccos(\frac{Tr(R_0^T R_t) - 1}{2})$$

result

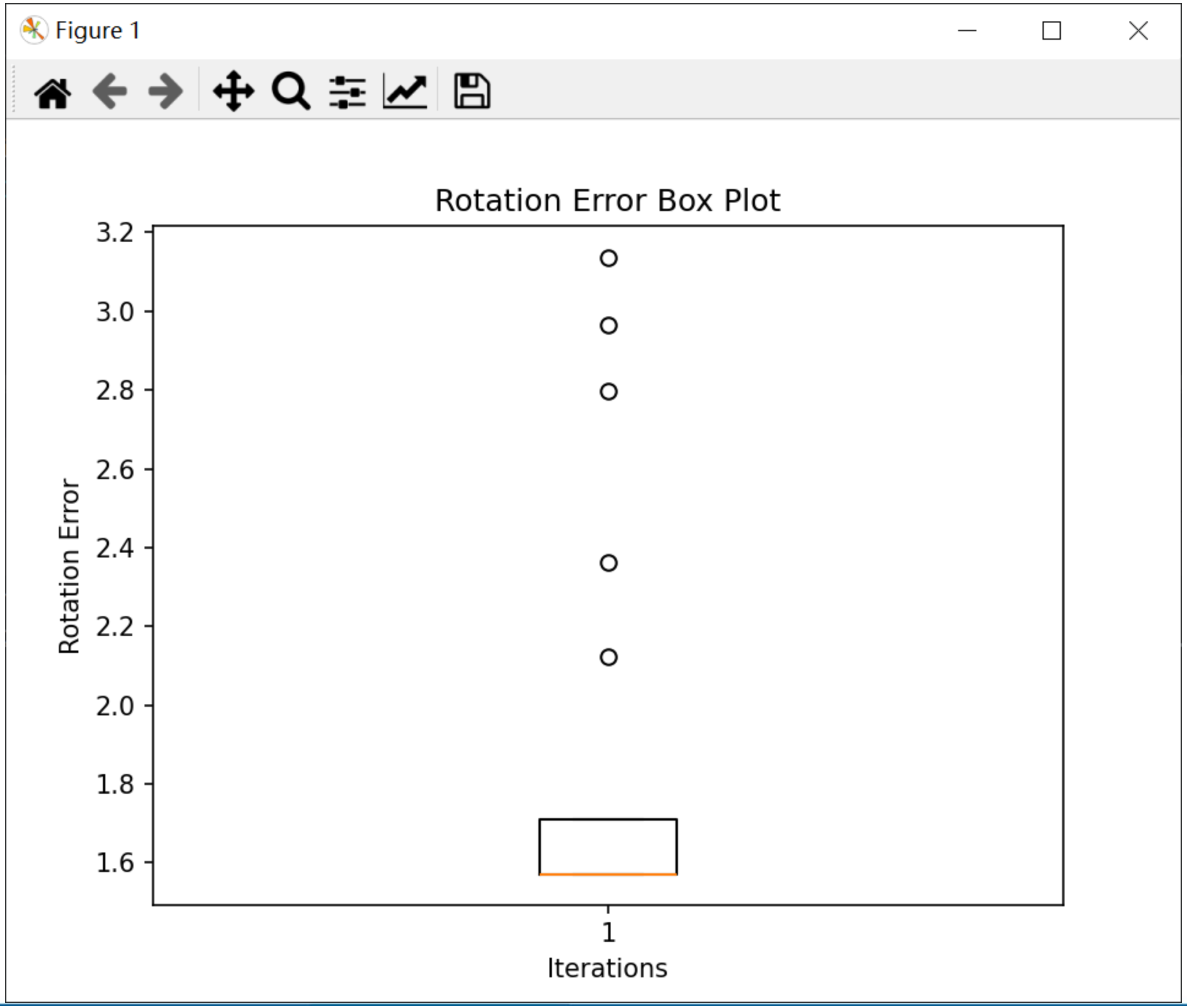
Table 1: Average errors in degrees—standard deviation—running times in seconds of various algorithms on synthetic data(20 trials)

Inlier Ratio	$10^3/10^5$	$10^3/10^6$	$3\times10^3/5\times10^6$	$3\times10^3/10^7$	$10^3/10^7$
TEASER++	out of memory				
RANSAC	0.39 0.2 29.1	$\geq$8.4 hours			
GORE	3.43 2.10 1698	$\geq$12 hours			
FGR	52.2 68.5 3.46	95.0 60.9 37.7	84.9 59.4 145	86.5 56.9 311	97.3 61.3 314
GNC-TLS	3.86 9.51 0.13	63.4 50.5 2.26	49.9 31.1 15.9	90.2 45.6 40.1	120 34.3 36.3
ARCS+R	9.92 13.1 0.12	65.2 48.9 0.96	55.6 38.3 5.58	88.4 36.2 12.6	98.2 36.0 12.2
ARCS+O	0.86 0.29 1.71	0.99 0.37 23.2	0.91 0.30 125	0.98 0.42 287	55.6 60.9 281
ARCS+OR	0.03 0.03 1.72	0.09 0.07 23.2	0.11 0.07 125	0.22 0.15 287	55.4 60.1 281
MINE	1.88 0.52 32.36	1.84 0.49 28.88	2.18 0.57 205.04	1.95 0.54 255.64	1.91 0.50 343.38

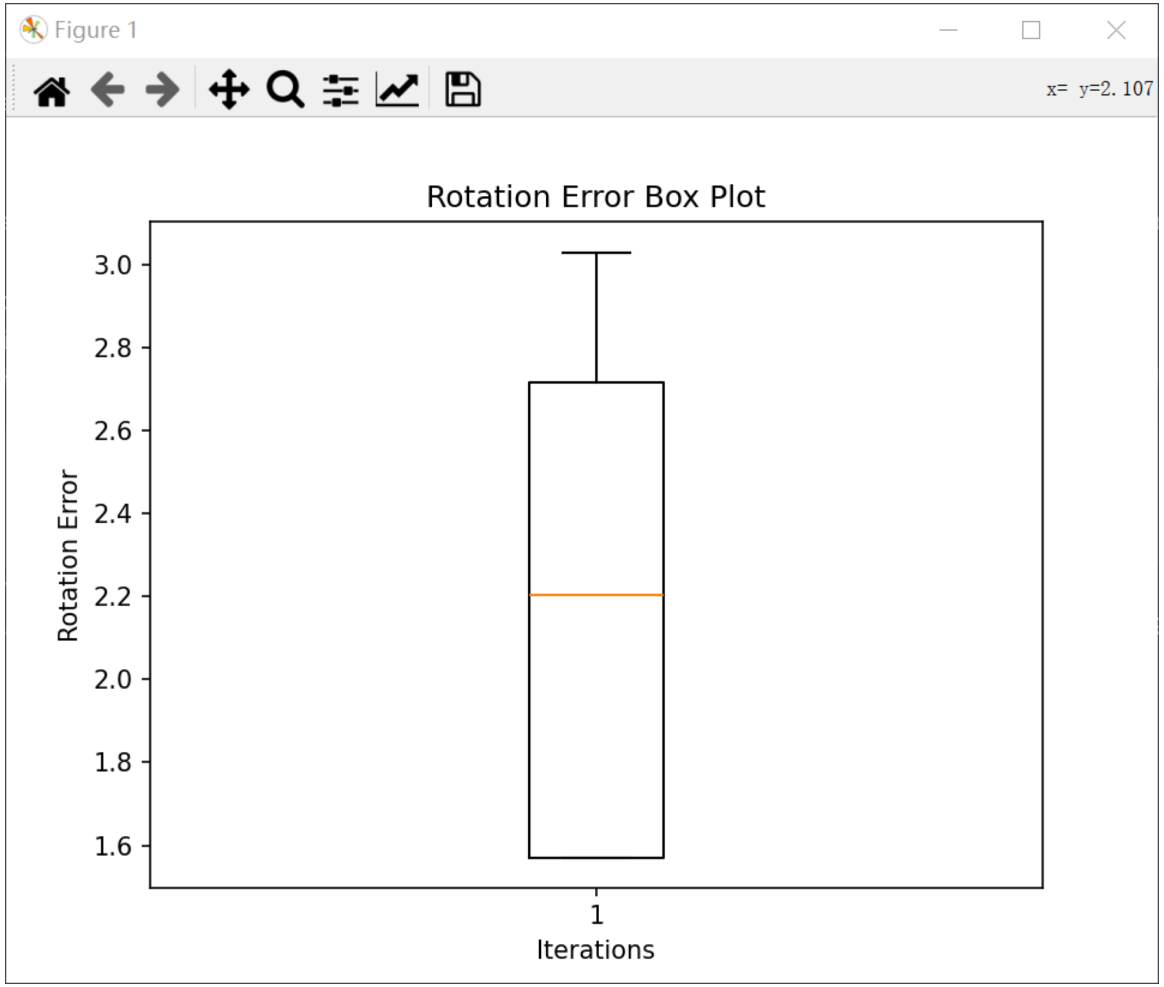
Result(inlier ratio=$\frac{10^3}{10^5}$)



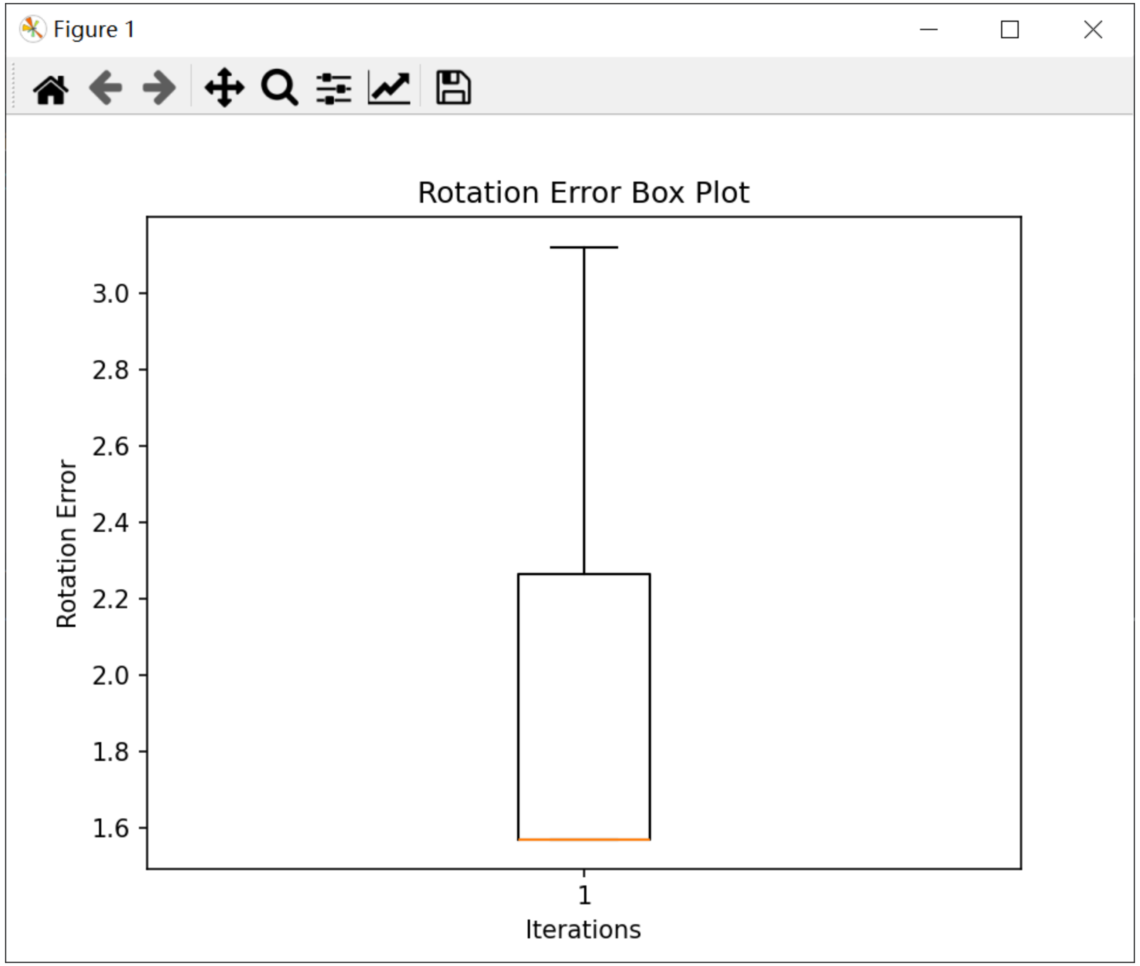
Result(inlier ratio=$\frac{10^3}{10^6}$)



Result(inlier ratio=$\frac{3\times10^3}{5\times10^6}$)



Result(inlier ratio=$\frac{3\times10^3}{10^7}$)



Result(inlier ratio=$\frac{10^3}{10^7}$)

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Thank You!

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